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Review for Exam II , MTH 221 , Fall 2010

Ayman Badawi

QUESTION 1. Are $(2,1),(-1,4),(6,9)$ independent in R^2? Explain

QUESTION 2. Let $F = span\{(1,1,-1,-1),(1,-1,2,0),(5,-1,4,-2)\}$. Find $dim(F)$. Find a basis B for F.
Is $(4,0,1,-1) \in F$? explain. Is $a = (3,-1,3,-1) \in F$? explain. If yes, then write a as a linear combination of the elements in B.

QUESTION 3. Let $D = span\{x^2+3x-1, -2x^2, x^2+x+1\}$. Find $dim(D)$. Find a basis B for D. Is $a = 5x+12 \in D$? explain. If yes, then write a as a linear combination of the elements in B.

QUESTION 4. Form a basis B for P_5 such that B containing the two elements $4x^4$, x^4+x and each element in B is of degree 4.

QUESTION 5. Let $M = span\{\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0.5 \end{bmatrix}, \begin{bmatrix} 5 & -3 \\ 0 & 2.5 \end{bmatrix}\}$. Find $dim(M)$. Find a basis B for M. Is $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \in M$? explain. If yes, then write A as a linear combination of the elements in B.

QUESTION 6. Let $A = \begin{bmatrix} -1 & 1 & -1 & 1 & 2 \\ 1 & -1 & 2 & -1 & -2 \\ -3 & 3 & -2 & -3 & -6 \end{bmatrix}$ Find a basis for $N(A)$, basis for $Col(A)$, and basis for $Row(A)$.

QUESTION 7. Let $B = \{(1,1,1,1),(-2,-1,2,1)\}$ be a basis of a vector space F. Describe F. Where does F "live"?. Given $v \in F$. Prove that there are unique numbers a, b such that $v = a(1,1,1,1) + b(-2,-1,2,1)$.

QUESTION 8. Let $F = \{\begin{bmatrix} 3a & 2b \\ 2a-b & 0 \end{bmatrix} \mid a,b \in R\}$. Is F a subspace of $R_{2\times 2}$? If yes, then (i) Find a basis for F and (ii) Write F as a span.

QUESTION 9. Let $F = \{3a + 2bx + (a-b)x^2 \mid a,b \in R\}$. Is F a subspace of P_3? If yes, then (i) Find a basis for F and (ii) Write F as a span.

QUESTION 10. Let $F = \{\begin{bmatrix} 3a & 2b \\ 2a-b & 1 \end{bmatrix} \mid a,b \in R\}$. Show that F is not a subspace of $R_{2\times 2}$.

QUESTION 11. Let $F = \{3a + 2bx + (a-b+2)x^2 \mid a,b \in R\}$. Show that F is not a subspace of P_3.

QUESTION 12. Let $F = \{(a,b,c,d) \mid a,b,c,d \in R$ and $2a - b + d = 0\}$. Show that F is a subspace of R^4. Find a basis for F. Write F as SPAN.

QUESTION 13. Let F, M be subspaces of R^{10} such that neither $F \subseteq M$ nor $M \subseteq F$. Prove that $F \cup M$ is never a subspace of R^{10}. Note that it is true that if M and F are subspaces of a vector space V, then $M \cap F$ is a subspace of V.

QUESTION 14. Let $F = \{(a,b,c,d) \mid a+2b+c+d = 0\}$ and $M = \{(a,b,c,d) \mid -a-b-c+d = 0\}$. LET $K = M \cap F$. Then K is a subspace of R^4. Find a basis for K. Write K as a span.

QUESTION 15. Let $T : P_3 \longrightarrow R$ be a linear transformation such that $T(p(x)) = \int_0^1 p(x)\, dx$. Find the standard matrix representation of T. Find $Ker(T)$. Write $Ker(T)$ as a SPAN.

QUESTION 16. Let $T : P_3 \longrightarrow R^3$ such that $T(1) = (2,2,2)$, $T(3x+6) = (-1,0,3)$, and $T(x^2) = (1,2,5)$. Find the standard matrix representation of T. Find a basis for $Ker(T)$ and a basis for Range(T)

QUESTION 17. Let $T : P_3 \longrightarrow R^3$ such that $T(f(x)) = (f(0), f(1), f(2))$.

(i) Show that T is a linear transformation. Then find the standard matrix representation of T.

(ii) Find basis for Ker(T) and basis for Range(T). Write Ker(T) and Range(T) as span.

(iii) Show that T is 1-1 and onto (isomorphism), and then define T^{-1}

QUESTION 18. Let $T : P_4 \longrightarrow R_{2\times 2}$ such that $T(f(x)) = \begin{bmatrix} f(1) & f(-1) \\ 0 & f(1) \end{bmatrix}$

(i) Show that T is a linear transformation.

(ii) Find the standard matrix representation of T.

(iii) Find basis for Ker(T) and basis for Range(T). Write Ker(T) and Range(T) as span.

(iv) Is T 1-1 ? Explain

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`